Out of Tune:
An Exploration of the Evolution of Tuning in Western Music

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Introduction:

In the current century, the complex history of musical tuning in the West is largely ignored by many musicians and composers. After all, the nearly universally adopted system of 12-tone equal temperament is highly functional, abundantly convenient, and has been widely used since the 19th century. However, the almost exclusive use of 12-tone equal temperament in Western music has caused many to forget the unique musical possibilities offered by other systems of tuning. The history of musical tuning is rich and complex, and its impact on music composition is extremely significant. In fact, the development of new tuning systems often inspired composers to write new music; conversely, it is also true that the evolution of the tonal harmonic system has necessitated the invention of new tuning systems. Hence, the influence of tuning on the progression of musical history is akin to that of technology: a behind-the-scenes factor linked to many more developments than commonly known. In this essay, I will explore the evolution of tuning in Western music, and how the specific characteristics of various tunings have influenced music composition.

Part 1: The Harmonic Series and Just Intonation:

The harmonic series is a series of overtones with frequencies corresponding to integer multiples of a fundamental frequency. Whenever a note is played on an instrument, the overtones of that note, though very difficult to hear, will be audible, and will influence the timbre of the sound. For instance, if a pianist plays the note C, this note acts as the fundamental and first partial of a harmonic series. The second partial, or first overtone, is the note C one octave above the fundamental, the third partial is the note G a twelfth above the fundamental, and the fourth and fifth partials correspond to C and E respectively. The harmonic series continues upward indefinitely; however, only the first few partials are audible to human ears.

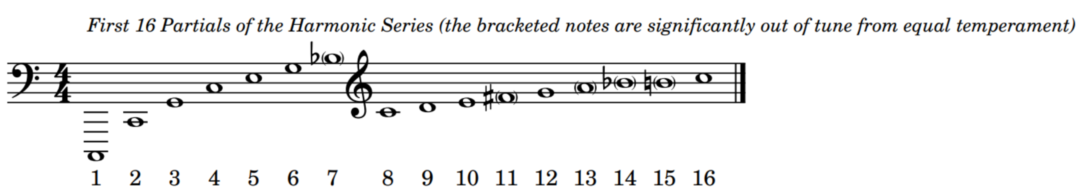


Fig. 1.1: Diagram of the Harmonic Series.

The importance of the harmonic series to music, besides its relation to timbre and chord voicing, is that it can be used to derive musical intervals. The frequency relationship between the fundamental or first partial and the first harmonic or second partial is 2:1. This 2:1 ratio represents the interval of an octave. The perfect fifth is derived from the 3:2 relationship between the third partial and the second partial, while the major third is derived from the 5:4 relationship between the fifth and fourth partials. The frequency ratios of these intervals are based on simple whole number ratios, and therefore have a simple, consonant sound. Because of their consonant sound, they have been used to construct a system of tuning called 5-limit just intonation.

The name 5-limit just intonation signifies that the frequencies of all twelve notes of the chromatic scale are obtained from a fundamental frequency by using some combination of the intervals of an octave, a perfect fifth, and a major third. For example, in a system with C as the fundamental frequency, the note F# can be obtained by going up two perfect fifths, and one major third (C to G, G to D, and then D to F#). The note Eb can be obtained by going down one major third, and then up one perfect fifth (C to Ab, Ab to Eb). As the 5:4 ratio of the major third is the most complex ratio used to derive other frequencies, and that ratio is based on the prime number five, the prefix 5-limit is added. The term just intonation describes any musical tuning system that uses whole number ratios to represent intervals; therefore, 5-limit tuning is a type of intonation tuning; however, just intonation systems based on more complex ratios, such as 7-limit tuning and 11-limit tuning, are also possible.

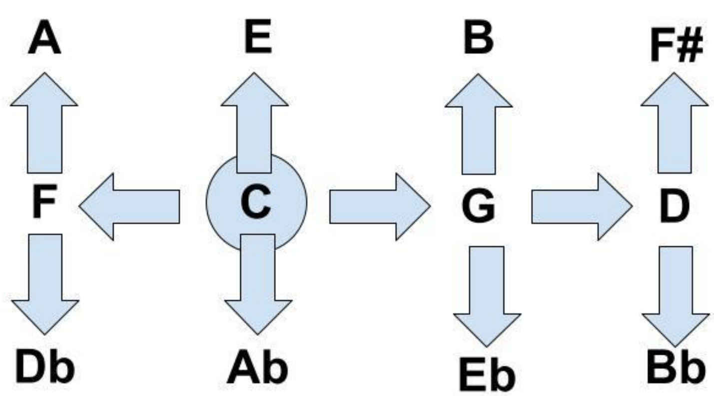


Fig. 1.2: Diagram Showing Derivation of Notes in 5-Limit Tuning, *(horizontal arrows represent derivation by upward or downward perfect fifth, vertical arrows represent derivation by upward or downward major third).*

Because of the fact that many of the intervals contained within just intonation systems have a very natural and consonant sound, many cultures around the world have used just intonation in their music. Accordingly, the first tuning systems used in the West were just intonation systems. For example, the first system of tuning known to have been theorized in the West was Pythagorean tuning, a system supposedly developed by Pythagoras during the 6th century BCE. This system of tuning could also be described as 3-limit just intonation. Later on, during the 2nd century CE, Roman mathematician Claudius Ptolemy developed a system of just intonation that closely resembled 5-limit tuning.

While it is true that the sound of just intonation is often described as “pure”, and that many of the chords and intervals it forms are endowed with an unparalleled quality of resonance; there are numerous practical difficulties associated with the use of just intonation. One of these difficulties is the phenomenon of “comma-pump”, an effect in which the pitch of a musical sequence rises each time it is repeated. This is due to the fact that the precise application of tuning math in certain sequences can cause the frequency of the fundamental pitch to increase by a small, but nevertheless significant amount. This discrepancy between two slightly different versions of the fundamental pitch is called a comma. Additionally, while just intonation systems are built on pure intervals, they also inevitably tend to contain a few intervals that are aggressively out of tune. These intervals are nicknamed “wolf intervals”, and they are almost completely unusable in the majority of musical contexts. Wolf intervals can be avoided, but this comes at the cost of musical creativity, as composers have to avoid writing music in certain keys in order to not feature such intervals.

Fig. 1.3: Diagram Showing “Comma-Pump”, *(the “+” accidentals indicate the gradual rising of the pitch).*

Part 2: Pythagorean Tuning:

Pythagorean tuning is a system of tuning, the development of which has been widely attributed to the Ancient Greek philosopher of the 6th century BCE, Pythagoras of Samos. However, some musicologists claim that the system was actually first developed by the Ancient Mesopotamians. According to legend, Pythagoras was walking past a forge when he heard the sound of several different hammers striking anvils simultaneously. Pythagoras noticed that the sound produced by the hammers was pleasing and consonant, and therefore decided to investigate why. He soon determined that the ratios obtained by comparing the weights of each of the hammers were simple integer ratios, such as 2:1 and 3:2, and then decided that these ratios were the key to beautiful music. Pythagoras believed that if the frequencies of musical pitches formed the same simple mathematical ratios that existed between the hammers, the result would be music that sounded harmonious, pleasing, and divine. This belief was consistent with the Pythagorean belief that the entirety of the universe was governed by mathematics and that numbers were divine and could be used to explain beauty.

Feeling inspired by his discovery, Pythagoras created a system of tuning that made use of mathematical ratios, and the result was Pythagorean tuning. Pythagorean tuning uses the 3:2 ratio of the pure perfect fifth as a way of deriving all the notes of the chromatic scale from a fundamental frequency. If the fundamental frequency used is the note C, the note G can be obtained by multiplying the frequency of C by 3:2. The note D can be obtained from G by doing the same multiplication, as can A from D, E from A, etc. This pattern continues until the note C is reached again at the end of the circle of fifths. The pitches attained by these multiplications can be shifted by octave by multiplying or dividing them by 2:1 (the ratio of an octave), and this can be done such that a one octave twelve-note chromatic scale is formed. Because the 3:2 ratio of a pure perfect fifth was used to derive all the notes of the scale, perfect fifths played in any key will have the same quality of beautiful resonance associated with the pure perfect fifth.

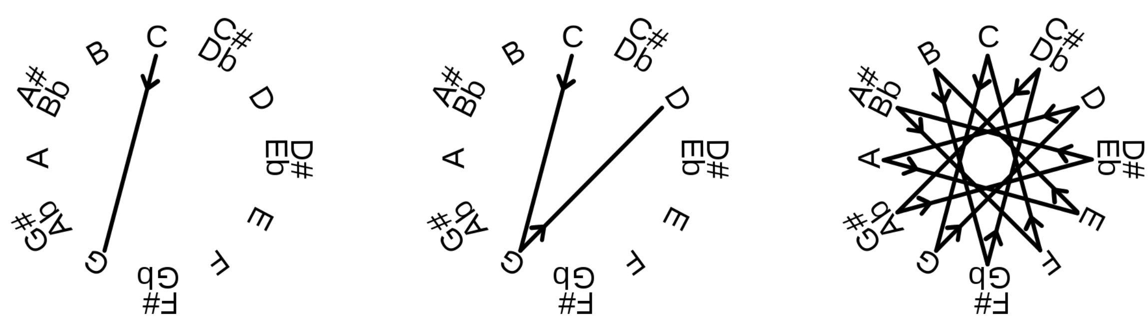
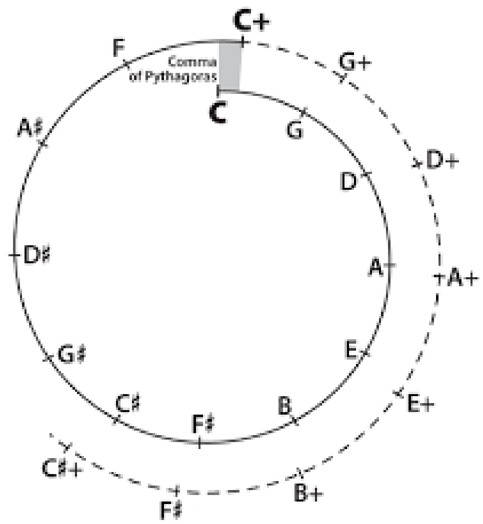


Fig. 2.1: Diagram Showing the Pythagorean Derivation of the Chromatic Scale Represented with the Circle of Fifths.

It seemed that Pythagoras had succeeded in creating a system of tuning that would allow for beautiful music in all twelve keys; however, this was not the case, because not all the perfect fifths in his chromatic scale were actually pure fifths. If a pianist were to play 12 consecutive ascending fifths on a piano tuned to 12 tone equal temperament, they would eventually arrive at a note exactly seven octaves above the note they started at. Conversely, going up seven fifths using the Pythagorean ratio of 3:2 would lead one to arrive at a note that is not exactly seven octaves above the note where they started, but that is actually 23 cents sharper (cents is a unit used for measuring pitch; one cent is equal to 1/100th of an equal-tempered semitone). Thus, it is impossible to have entirely pure fifths in a tuning system, because pure octaves would have to be sacrificed. Fundamentally, 7 octaves does not equal 12 fifths, and in the case of Pythagoras, this fact gives rise to something called the Pythagorean comma. This is an interval that can be defined as the difference between 7 octaves and 12 fifths where pure ratios are in use, and this translates to a gap of approximately 23 cents, or nearly a quarter of a semitone.



Beaulieu, John. Image from “The Perfect Fifth: The Science and Alchemy of Sound”. *The Rose+Croix Journal* Vol.11, https://www.rosecroixjournal.org/

Fig: 2.2: Diagram Showing the Chain of Fifths That Leads to the Pythagorean Comma.

The other implication of not being able to have entirely pure fifths in a tuning system is obvious: at least one of the fifths must be a different size. In Pythagorean tuning, the last fifth in the chain of 12, or F to C in a system where C is the fundamental tone, is made smaller by one Pythagorean comma, giving it a ratio of 262144:177147. As opposed to the simple 3:2 ratio of a just perfect fifth, this ratio is monstrously complex, and therefore produces an interval that sounds very dissonant. Because of its dissonance, this interval has been given the name “wolf interval”, as the two notes are in such discord that they seem to howl at each other like wolves.

The existence of this wolf interval was very problematic, because its dissonance made it essentially unusable in music. All that could be done was to move the interval to a place where it likely wouldn't be used often, such as between G# and Eb (the spelling is key to the derivation), and this was done by changing the derivation of some of the notes of Pythaogorean tuning such that some notes were reached by downward fifths instead of upward fifths. Although this did represent a partial solution to the problem of the wolf fifth, it also meant that certain keys couldn't be used for making music.

Besides the wolf fifth, Pythagorean tuning also presented other problems. Ordinarily, major thirds have the ratio of 5:4; however, in Pythagorean tuning most of the major thirds have a ratio of 81:64. This results in major thirds that are approximately 22 cents sharper than pure thirds, and therefore sound quite out of tune. The ramifications of this on music making were very significant. In the Middle Ages, thirds and sixths were considered dissonant intervals in part because Pythagorean tuning made them so. This in turn influenced music composition, because it meant that only octaves, fifths, and fourths could be used in harmony. Consequently, one can clearly observe the prevalence of using only octaves, fifths, or fourths to harmonize melodic lines in the middle ages. Eventually, new tuning systems were developed that fixed the problem of dissonant thirds and sixths, making possible the development of a new style of composition where these intervals could be used in harmony; however, Pythaogrean tuning was used by musicians up until the beginning of the 16th century, and therefore holds a sort of record as one of the longest used tuning systems in Western music.

Fig. 2.3: Opening bars of "Ave Maris Stella", by the medieval composer Perotin, *(note the prevalence of octaves, fifths, and fourths in the harmony).*

Part 3: Tempered Tuning Systems:

Tempered tuning systems are those tuning systems in which the sizes of certain intervals have been altered in order to respond to some of the practical problems associated with using just intonation. In the West, many of the first tempered tuning systems to be invented were systems that sought to rectify the impurity of thirds and sixths in Pythagorean tuning. This development was mostly brought about by the evolution of the harmonic style of Western music, which saw the introduction of the triad, to which harmony in thirds and sixths is essential. Therefore, the inventors of the earliest tempered tuning systems abandoned the Pythagorean goal of having as many pure fifths as possible, and replaced it with the goal of having as many pure thirds and sixths as possible. Hence, the pure fifths had to be tempered, meaning that their size had to be altered by a certain amount. The tuning systems that were designed to accomplish this goal are known today as meantone temperaments, and while many different meantone temperament systems were invented, the most popular was quarter-comma meantone.

Quarter-comma meantone temperament was first described by Italian music theorist Pietro Aron in his 1523 book, *Toscanella de la Musica*, and the system is so named because it uses a pure perfect fifth that has been flattened by one quarter of a syntonic comma. This slightly narrower version of a perfect fifth no longer has a ratio of 3:2, and can actually be described by the value $\sqrt[4]{5}$**,** which is approximately equivalent to 697 cents, while the 3:2 fifth corresponds to approximately 702 cents. As with Pythagorean tuning, one can derive all the notes of quarter-comma meantone temperament by multiplying a fundamental frequency by the value that is used for a fifth. In this instance, to go up by 12 fifths (thereby getting a value for each note of the chromatic scale), one would have to raise $\sqrt[4]{5}$ to the power of 12.

However, as with Pythagorean tuning, this value is not equal to going up by seven octaves. In fact, it falls short by approximately 41 cents, meaning that the last fifth in the chain will be a wolf fifth that is 41 cents sharper than all the other fifths in the system. Thus, quarter-comma meantone temperament faces the same problem that plagues Pythagorean tuning. However, it still represents an improvement upon Pythagorean tuning, as the system features eight pure major thirds with a ratio of 5:4, while Pythagorean tuning features no pure major thirds. Because of this, quarter-comma meantone temperament can be used to construct triads that don't sound horribly out of tune, making it suitable for performing triad-based tonal music. The system does still have limitations, as only a few keys have usable triads, but as long as music played using quarter-comma meantone temperament is written in a favorable key, and doesn't modulate to any unfavorable keys, it will sound sufficiently pleasant and consonant.

	Number of	Width (in cents) of intervals starting from...											
	semitones	D	E♭	E	F	F♯	G	G♯	A	B♭	B	C	C♯
Unison	0	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
Minor second	1	117	76	117	76	117	76	117	117	76	117	76	117
Major second	2	193	193	193	193	193	193	234	193	193	193	193	234
Minor third	3	310	269	310	269	310	310	310	310	269	310	310	310
Major third	4	**386**	**386**	**386**	**386**	427	**386**	427	**386**	**386**	427	**386**	427
Perfect fourth	5	503	462	503	503	503	503	503	503	503	503	503	503
Augmented fourth	6	579	579	621	579	621	579	621	621	579	621	579	621
Perfect fifth	7	697	697	697	697	697	697	738	697	697	697	697	697
Minor sixth	8	814	773	814	773	814	814	814	814	773	814	773	814
Major sixth	9	890	890	890	890	931	890	931	890	890	890	890	931
Minor seventh	10	1007	966	1007	1007	1007	1007	1007	1007	966	1007	1007	1007
Major seventh	11	1083	1083	1124	1083	1124	1083	1124	1083	1083	1124	1083	1124
Octave	12	**1200**	**1200**	**1200**	**1200**	**1200**	**1200**	**1200**	**1200**	**1200**	**1200**	**1200**	**1200**

Fig. 3.1: Diagram Showing the Width of Intervals in Quarter-Comma Meantone, *(the values highlighted in gray are significantly out of tune).*

Spitta, Philipp and Seiffert, Max (Editors). Buxtehude, Dieterich (Composer). "Ciacona in C Minor: BuxWV 159". *Orgelcompositionen, Band I, No.2* (pp.6). 1903). Public Domain.

Fig. 3.2: Excerpt from Buxtehude's "Ciacona in C Minor: BuxWV 159", an organ composition of the mid-Baroque era, *(note the prevalence of harmony by thirds).*

Though quarter-comma meantone temperament and other types of meantone temperament were popular throughout much of the late Renaissance and Baroque eras, the limitations of these tuning systems were still apparent to musicians. These limitations began to pose a more significant problem as the tonal harmonic system further developed, since composers wanted to write music in more than just a few keys, and wanted to have the liberty to modulate to any key they desired. As a result of this, the music theoreticians and mathematicians of the 17th and 18th centuries worked to develop new tuning systems that would allow for greater musical freedom. The result of their labors was the invention of a new type of tempered tuning called well temperament. The term well temperament doesn't refer to a specific tuning system, but to any tuning system in which the 12 notes of the chromatic scale are tuned such that music can be played in all major and minor keys without sounding very out of tune.

Though there are many tuning systems that can be classified as well temperament, one of the most popular today is Werckmeister III, a system invented by German music theorist and composer, Andreas Werckmeister, and first described in his 1691 book, *Musikalische Temperatur*. Unlike Pythagorean tuning and quarter-comma meantone temperament, which both have 11 fifths of the same size and one highly dissonant wolf fifth, Werckmeister III has four tempered fifths and eight pure fifths. In a system with C as the fundamental frequency, the four tempered fifths are C to G, G to D, D to A, and B to F#, and they have each been tempered by one quarter of a Pythagorean comma (the interval that represents the difference between a stack of 12 pure fifths, and a stack of 7 octaves). The rest of the fifths in Werckmeister III are pure.

Note	C	C#	D	Eb	E	F	F#	G	Ab	A	Bb	B
Cents	0	90	192	294	390	498	588	696	792	888	996	1092

Fig. 3.3: Table Showing Values for Werckmeister III in Cents, *(note how close many of the values are to equal temperament).*

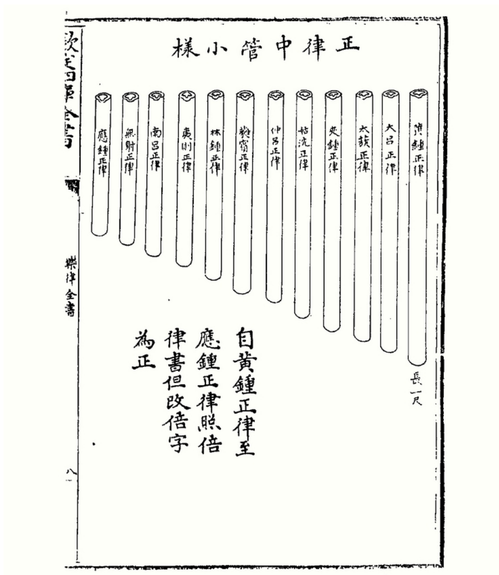
In essence, Werckmeister III and other well temperament systems responded to the problem of many keys being unusable in meantone temperament by making small calculated sacrifices to the purity of multiple intervals. Because these sacrifices were small, and because they were made fairly evenly throughout the chromatic scale, none of the 24 major and minor keys sound significantly out of tune. Additionally, most well tempered tuning systems still preserved many pure intervals. As a result of these characteristics, well temperament not only made music possible in all major and minor keys, but it also afforded a slightly different character and color to each key, as the sizes of certain intervals varied slightly in each key.

For musicians and composers, well temperament represented a practical and liberating solution to the problem of tuning, and many musicians and composers would exploit the possibilities of the system to write music that would not have been possible with earlier tuning systems. One very notable example is Johann Sebastain Bach's two sets of preludes and fugues in all 24 major and minor keys, titled *The Well-Tempered Clavier*. Although there is debate concerning exactly what tuning system Bach wanted performers to use for *The Well-Tempered Clavier*, most musicologists agree that it was some kind of well-tempered tuning system, with some even arguing for a system very similar to Werckmeister III. The invention of well tempered tuning inspired developments in the tonal harmonic system, as it allowed composers to experiment with increased chromaticism in their music, while also making possible more unusual modulations. Though well temperament would eventually be replaced by equal temperament, it remains an important step in the evolution of tuning.

Fig. 3.4: An Excerpt from Bach's "Prelude and Fugue in Bb minor: BWV 891", a late-Baroque keyboard work, *(note the greater use of non-diatonic tones, and the key signature of Bb minor, a key whose practical use was enabled by well temperament).*

Part 4: 12-Tone Equal Temperament:

12-tone equal temperament is a musical tuning system in which the octave is divided into 12 equally spaced parts. Though 12-tone equal temperament was not calculated with mathematical exactitude until 1584 by Chinese mathematician and music theorist Zhu Zaiyu, it had been theorized and discussed much earlier, by many different historical figures, such as Greek philosopher Aristoxenus, and Italian astronomer Galileo. In the West, equal temperament was first calculated exactly by Flemish mathematician Simon Stevin in 1585; however, at that point, it had already been in use in fretted instruments such as the lute, which wasn't particularly suited to any other tuning system.



16th century scientist Zhu Zaiyu, Public domain, via Wikimedia Commons

Fig. 4.1: An Image of Zhu Zaiyu's Manuscript Showing his Calculation of 12-Tone Equal Temperament.

Despite this, equal temperament was not widely adopted in the musical community until the late 18th century. This is because many music theorists were opposed to equal temperament, claiming that it ruined the purity of all intervals, and therefore made music sound significantly out of tune. There is some truth to this, as equal temperament represents a sort of ultimate compromise in tuning, sacrificing the purity of all intervals but the octave for unrestricted harmonic freedom. Notwithstanding this resistance, in the latter half of the 18th century, equal temperament began to gain popularity, not simply because its supporters desired harmonic freedom, but also because the simplicity of its design made it convenient to implement. By the early Romantic Era, the use of equal temperament became standard in most musical communities, and its predominance was aided by the fact that instruments using equal temperament, such as the piano, could now be mass produced, and therefore made available to many musicians of different social classes.

Compared to other tuning systems, 12-tone equal temperament is much simpler to construct. In 12-tone equal temperament, all semitones are exactly the same size, and can therefore all be represented with the value $\sqrt[12]{2}$, which is an exact mathematical description of the division of the octave (which has a ratio of 2:1, or just 2) into an interval one twelfth of its size. Since all semitones are the same size in equal temperament, all fifths and thirds, and all other intervals are also the same size, regardless of where one is in the chromatic scale. There are both advantages and disadvantages that result from tuning instruments in this manner. First, it makes possible the transposition of any piece of music into any key, while also allowing composers to make use of whatever modulations they want in their music. Equal temperament also allowed for the development of more complex harmonic structures, such as those found in jazz, and in classical music of the 20th and 21st centuries. In particular, harmonic approaches such as polytonality, modal harmony, secundal harmony, non-functional harmony, and freely atonal harmony simply wouldn't have been possible without equal temperament. Equal temperament is also essential to 12-tone serialism, a compositional technique invented by Arnold Schoenberg which seeks to achieve compositional equality between all 12 tones of the chromatic scale.

Fig. 4.2: An Excerpt from Schoenberg's "Three Piano Pieces No.1: Op.11", a 20th century piano work, *(note the use of 10 out of 12 notes of the chromatic scale, a level of chromaticism facilitated by equal temperament).*

Despite the many advantages of equal temperament, not everyone views it favorably. Some people believe that equal temperament sounds cold, sterile, and mechanical, while others argue that many of its intervals simply don't sound very good. This is because all the intervals in equal temperament (except the octave) deviate to some degree from their pure counterparts. For example, equal-tempered major thirds are 14 cents sharp, and equal-tempered minor thirds are 16 cents sharp. Consequently, some even claim that equal temperament has ruined harmony, while others are not pleased with the fact that equal temperament doesn't allow for each key to have a unique character, as they did in well temperament.

Notwithstanding this, equal temperament has been universally adopted across the Western world, so much so that many pure intervals sound somewhat out of tune to the modern ear. Although equal temperament is technically out of tune compared to just intonation, the average listener does not notice this, because they have likely only ever heard music in equal temperament. Similarly, the average musician has very little knowledge of any other tuning system besides equal temperament, as its predominance in the world, and its musical flexibility gives them little reason to wonder if there may be a better way to approach the tuning of musical instruments. It seems that with equal temperament, the West has completed its millennia long quest to solve the problem of musical tuning. Indeed, the versatility of equal temperament will likely assure its dominance in the future; however, it is a shame that this dominance leaves little room for further experimentation in the field of tuning.

Part 5: Modern Use of Just Intonation:

Despite the predominance of 12-tone equal temperament in music of the 20th and 21st centuries, the history of tuning certainly did not end with the widespread adoption of that particular system by musicians and composers of the West. In fact, a small number of composers, musicians, and music theorists have continued to experiment with the possibilities of musical tuning. Although there are some who may believe that equal temperament should be replaced by a new system of tuning, the majority of people who have investigated tuning in the time since the adoption of equal temperament have not done so with the goal of designing such a system. Rather, their exploration has been based on the premise of finding new and unique soundworlds, and expressing musical ideas in an innovative way. This sentiment is especially exemplified by composers who write music specifically with the goal of showcasing and considering the possibilities of different tuning systems.

One such example is La Monte Young, an American composer born in 1935 who was one of the first minimalist composers. His crowning compositional achievement is a work for retuned piano called *The Well-Tuned Piano*: an incredible improvisatory composition first performed in 1974, that utilizes 7-limit just intonation tuning. Since its first performance, the length of *The Well-Tuned Piano* has grown significantly, lasting over six hours in a version recorded in 1987. Young's tuning for the piece is quite unique, as it uses both the just perfect fifth, an interval with a ratio of 3:2, and the harmonic seventh, an interval with a ratio of 7:4 (which is based on the prime number 7, and therefore makes Young's tuning a type of 7-limit just intonation), to derive all 12 notes of the chromatic scale.

The various intervals to which this tuning system gives rise, such as the septimal minor third and the septimal major third, are quite distinct and idiosyncratic, yet highly beautiful. Indeed, none of the tuning systems previously discussed in this essay feature any of them. Because of this, some of the harmonies used by Young sound quite otherworldly. However, they are also incredibly exhilarating, and even heavenly to the open ear. In *The Well-Tuned Piano*, La Monte Young has certainly found a very unique mode of musical expression, as well as a marvelous soundworld.

Piano key	12-TET (cents)	*Well-Tuned Piano* (cents)	Ben Johnston's notation	Interval name
E♭	0.00	0.00	E♭	Unison
E♮	100.00	176.65	F7++	567th harmonic
F	200.00	203.91	F+	Major tone
F♯	300.00	239.61	G77♭+	147th harmonic
G	400.00	470.78	A7♭+	21st harmonic
G♯	500.00	443.52	A77♭++	1323rd harmonic
A	600.00	674.69	B7♭+	189th harmonic
B♭	700.00	701.96	B♭	Perfect fifth
B♮	800.00	737.65	C77♭+	49th harmonic
C	900.00	968.83	D7♭	Harmonic 7th
C♯	1000.00	941.56	D77♭+	441st harmonic
D	1100.00	1172.74	E7♭+	Inverse septimal comma
E♭	1200.00	1200.00	E♭	Octave

Fig. 5.1: Table Comparing Cents Values for "The Well-Tuned Piano", and Equal Temperament, *(note the very significant differences between the two tuning systems, and the very unusual interval names).*

Another example is Ben Johnston, an American composer who lived from 1926 to 2019. Johnston is considered one of the most important composers of microtonal and just intonation music in the 20th century, designing his own microtonal notation system, and writing many works that thoroughly explored the massive possibilities of just intonation as a compositional approach. Instead of solely using relatively simple forms of just intonation, such as the 5-limit and 7-limit variants, Johnston sometimes employed 13-limit, 19-limit, and even 31-limit just intonation tuning. Thus, many of his pieces divide the octave into much more than 12 tones. For example, Johnston's Sixth String Quartet requires 61 divisions of the octave.

This type of compositional approach produces many complex intervals that are completely unknown to virtually all listeners, meaning that his music can sound somewhat uncanny and alien. However, some of Johnston's music is surprisingly accessible, such as his Fourth String Quartet, which is based on the Christian hymn "Amazing Grace". Overall, Johnston's work is united by a theme of fearless harmonic exploration, and of approaching tuning as a compositional device akin to rhythm, melody, and dynamics. This makes his music extremely interesting to listen to, yet exceedingly difficult to perform. Despite this, Johnston's music has not been ignored by performers, and many recordings of his works exist which succeed in accurately conveying the intricacy of his approach to tuning.

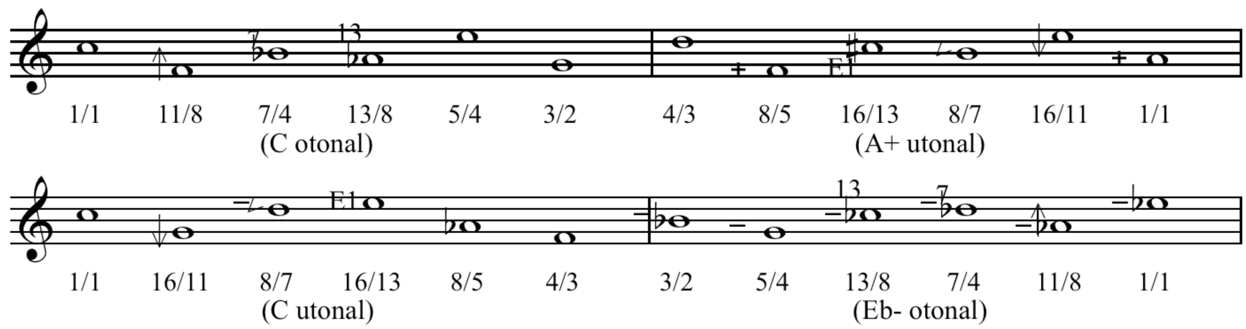


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Fig. 5.2: Tone Row Used in Ben Johnston's "String Quartet No.7", *(note the use of just intonation intervals, and the presence of intervals constructed from prime factors 11 and 13).*

The compositional outlooks of these two composers exemplify the manner in which composers of the 20th and 21st centuries can utilize tuning to write unique and extraordinary music that communicates equally unique and extraordinary ideas. Indeed, just as harmony, melody, rhythm, timbre, articulation, and dynamics are all musical devices which composers are expected to be able to manipulate in order to produce a desired compositional effect, it has been shown that tuning may be similarly manipulated. Thus, tuning has begun to influence music composition in a new way. The composer is no longer required to adhere to the default tuning of their time, as it is in their power to select whatever system of tuning they want, based on how they want their music to sound. In other words, the music will no longer serve the tuning; the tuning will serve the music. One can only hope that in the future, composers, musicians, listeners, and theorists will continue to experiment with new systems of tuning.

Conclusion:

The story of the evolution of tuning in Western music may have begun as a search for perfection in an imperfect world, yet it soon evolved into a search for compromise based on practical ideals. Since then, it has evolved once more into a search for creative possibilities. Throughout each stage of its development, tuning has never been an isolated science, but an art that has maintained an inseverable connection with music performance and composition. Though other factors may have had a more obvious influence on the progressions of musical history, music and tuning have consistently evolved side-by-side, directly impacting each other's transformation. Indeed, the specific characteristics of a given tuning system have always influenced the compositional process, as these characteristics are themselves an element of the music.

For this reason, I wish that more music lovers had an understanding of the history of tuning, as it would surely lead to a greater understanding of music in general. On a personal note, I love learning about the complexities of various tuning systems, and tuning is a subject that has consistently fascinated me. Though I greatly enjoy listening to music that doesn't use 12-tone equal temperament, I often have a hard time finding such music, because its abundance in the musical world is quite limited. Therefore, I write this essay in the hopes that more music-lovers will become fascinated with tuning, and that this will lead to the existence of a larger body of music that ventures outside of equal temperament.

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